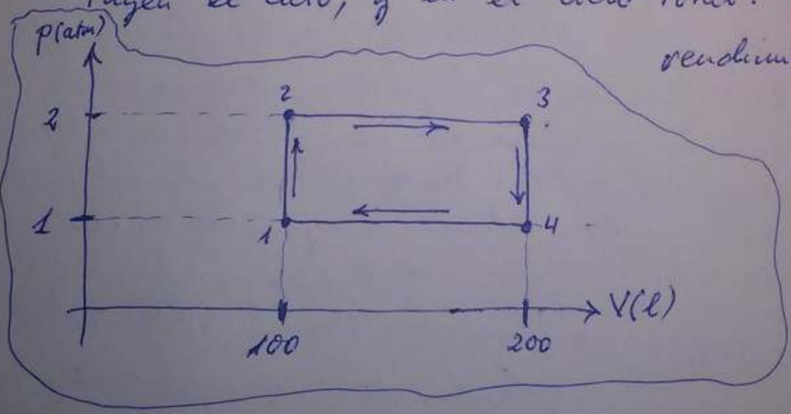
(3) Dos moles de un gas ideal ($c_v = 3$ cal/mol·K) describen el ciclo de la figura, determinar la temperatura de cada vértice y el trabajo, calor, variación de energía interna y variación de entropía en cada una de las líneas que constituyen el ciclo, y en el ciclo total. Determinar el rendimiento del ciclo.



Solución:

Criterio de signos: trabajo W o calor Q que salga del sistema: NEGATIVO

$P_2=P_3=2\,atm$; $V_3=V_4=200\,L$; $n=2$ moles ; GAS PERFECTO: $pV=nRT$

$P_1=P_4=1\,atm$; $V_1=V_2=100\,L$; $C_v=3\,\frac{cal}{mol\cdot K}$; $R=0{,}082057\,\frac{atm\,L}{mol\,K}=1{,}9872\,\frac{cal}{mol\,K}$

* Transformación isócora ①→② : $V_1=V_2=100\,L=cte$

$P_1V_1=nRT_1 \therefore T_1=\frac{P_1V_1}{nR}=\frac{1\,atm\cdot 100\,L}{2\,moles\cdot 0{,}082057\,\frac{atm\,L}{mol\,K}} \therefore T_1=609{,}332537\,K$ $\boxed{T_1=609\,K}$

$\frac{P}{T}=\frac{nR}{\underset{cte}{V}}=cte \therefore \frac{P_2}{T_2}=\frac{P_1}{T_1} \therefore T_2=\frac{P_2}{P_1}T_1=\frac{2\,atm}{1\,atm}\cdot 609{,}332537=1218{,}66507\,K \therefore$

$\therefore \boxed{T_2=1219\,K}$; $C_v=\frac{1}{n}\left(\frac{\delta Q}{\partial T}\right)_v=\frac{1}{n}\left(\frac{\partial U}{\partial T}\right)_v \therefore \int_1^2 dU=nC_v\int_{T_1}^{T_2}dT \therefore \Delta U_{12}=nC_v\Delta T_{12} \therefore$

$\therefore \Delta U_{12}=U_2-U_1=nC_v(T_2-T_1)=2\,moles\cdot 3\,\frac{cal}{mol\cdot K}(1218{,}66507-609{,}332537)\,K=$

$=3.655{,}9952\,cal \therefore \boxed{\Delta U_{12}=3656\,cal}$ 1er principio de la termodinámica: $\Delta U_{12}=Q_{12}+W_{12}$

$W_{12}=-\int_1^2 P\,\overset{0\,(isócora)}{dV} \Rightarrow \boxed{W_{12}=0} \therefore Q_{12}=\Delta U_{12} \therefore \boxed{Q_{12}=3656\,cal}$; $dS=\frac{đQ}{T}=\frac{dU}{T}=nC_v\frac{dT}{T} \therefore$

$\therefore \int_1^2 dS=S_2-S_1=\Delta S_{12}=nC_v\int_{T_1}^{T_2}\frac{dT}{T}=nC_v\ln\frac{T_2}{T_1}=2\,moles\cdot 3\,\frac{cal}{mol\cdot K}\cdot\ln\frac{1218{,}66507}{609{,}332537}=$

$=4{,}1588831\,\frac{cal}{K} \therefore \boxed{\Delta S_{12}=4{,}16\,\frac{cal}{K}}$

* Transformación isóbara ②→③ : $P_2=P_3=2\,atm=cte=P$; $\frac{V}{T}=\frac{nR}{\underset{cte}{P}}=cte$; $\frac{V_2}{T_2}=\frac{V_3}{T_3} \therefore$

$\therefore T_3=\frac{V_3}{V_2}T_2=\frac{200\,L}{100\,L}\cdot 1218{,}66507=2437{,}33014\,K \Rightarrow \boxed{T_3=2437\,K}$

$\Delta U_{23}=nC_v(T_3-T_2)=2\,moles\cdot 3\,\frac{cal}{mol\,K}(2437{,}33014-1218{,}66507)\,K=7311{,}9904\,cal$

$\boxed{\Delta U_{23}=7312\,cal}$ $W_{23}=-\int_2^3 p\,dV=-p(V_3-V_2)=p(V_2-V_3)=nRT_2-nRT_3=$

$=nR(T_2-T_3)=2\,moles\cdot 1{,}9872\,\frac{cal}{mol\,K}(1218{,}66507-2437{,}33014)\,K=-4843{,}4625\,cal$

$\boxed{W_{23}=-4843\,cal}$ 1er principio $\Rightarrow Q_{23}=\Delta U_{23}-W_{23}=7311{,}9904-(-4843{,}4625)=$

$=12.155{,}4529\,cal \therefore \boxed{Q_{23}=12155\,cal}$ $dS=\frac{đQ}{T}=\frac{dU-\overset{-p\,dV}{đW}}{T}=nC_v\frac{dT}{T}+\frac{\overset{nR/V}{P}}{T}dV$

$\Delta S_{23}=S_3-S_2=nC_v\int_{T_2}^{T_3}\frac{dT}{T}+nR\int_{V_2}^{V_3}\frac{dV}{V}=nC_v\ln\frac{T_3}{T_2}+nR\ln\frac{V_3}{V_2}=nC_v\ln\frac{T_3}{T_2}+$

$+nR\ln\frac{\frac{\cancel{nR}T_3}{\cancel{P_3}}}{\frac{\cancel{nR}T_2}{\cancel{P_2}}}=n\underbrace{(C_v+R)}_{\substack{C_p\\ \text{relación de Mayer}}}\ln\frac{T_3}{T_2}=2\,moles\cdot(3+1{,}9872)\,\frac{cal}{mol\,K}\ln\frac{2437{,}33014\,K}{1218{,}66507\,K}=$

$=6{,}91372724\,\frac{cal}{K} \therefore \boxed{\Delta S_{23}=6{,}914\,\frac{cal}{K}}$

* Transformación isócora ③ → ④ : $V_3 = V_4 = 200\,L = cte$; $\frac{P_3}{T_3} = \frac{P_4}{T_4}$; $T_4 = \frac{P_4}{P_3} T_3 =$

$= \frac{1\,atm}{2\,atm} \cdot 2437{,}33014\,K = 1218{,}66507\,K \therefore \boxed{T_4 = 1219\,K}$ ¡$= T_2$!

$\Delta U_{34} = n\,C_v (T_4 - T_3) = 2\,mol \cdot 3\,\frac{cal}{mol \cdot K} (1218{,}66507 - 2437{,}33014)\,K = -7311{,}9904\,cal$

$\boxed{\Delta U_{34} = -7312\,cal}$ ¡$= -\Delta U_{23}$! ; $\boxed{W_{34} = 0}$ $\Delta S_{34} = n\,C_v \ln \frac{T_4}{T_3} = 2\,mols \cdot 3\,\frac{cal}{mol\,K} \ln \frac{1218{,}66507\,K}{2437{,}33014\,K} =$

$= -4{,}1588831\,cal \therefore \boxed{\Delta S_{34} = -4{,}16\,\frac{cal}{K}}$ ¡$= -\Delta S_{12}$!

$\boxed{Q_{34} = -7312\,cal}$

* Transformación isóbara ④ → ① : $P_1 = P_4 = 1\,atm = cte = P$

$\Delta U_{41} = n\,C_v (T_1 - T_4) = 2\,mol \cdot 3\,\frac{cal}{mol \cdot K} (609{,}332537 - 1218{,}66507)\,K = -3655{,}9952\,cal$

$\boxed{\Delta U_{41} = -3656\,cal}$ ¡$= -\Delta U_{12}$! $W_{41} = nR(T_4 - T_1) = 2\,mols \cdot 1{,}9872\,\frac{cal}{mol \cdot K} (1218{,}66507 - 609{,}332537)\,K$

$W_{41} = 2421{,}7312\,cal \therefore \boxed{W_{41} = 2422\,cal}$ 1er principio de la termo $\Rightarrow Q_{41} = \Delta U_{41} - W_{41} =$

$= -3655{,}9952 - 2421{,}7312 = -6077{,}726\,cal \therefore \boxed{Q_{41} = -6078\,cal}$

$\Delta S_{41} = n(C_v + R) \ln \frac{T_1}{T_4} = 2\,mols\,(3 + 1{,}9872)\,\frac{cal}{mol \cdot K} \ln \left(\frac{1218{,}66507\,K}{609{,}332537\,K} \right)^{-1} = -6{,}9137272\,\frac{cal}{K}$

$\boxed{\Delta S_{41} = -6{,}914\,\frac{cal}{K}}$ ¡$= -\Delta S_{23}$!

* Ciclo total: obviamente para las magnitudes termodinámicas que son función de estado, su variación es nula en el ciclo total. Esto ocurre para ΔU y ΔS ∴ $\Delta U = U_1 - U_1 = 0$ y $\Delta S = S_1 - S_1 = 0$ ∴

∴ $\boxed{\Delta U_{ciclo} = 0 \quad \Delta S_{ciclo} = 0}$ Comprobación : $\Delta U_{ciclo} = \underbrace{3656}_{\Delta U_{12}} + \underbrace{7312}_{\Delta U_{23}} + \underbrace{(-7312)}_{\Delta U_{34}} + \underbrace{(-3656)}_{\Delta U_{41}} = 0$ #

$\Delta S_{ciclo} = \underbrace{4{,}16}_{\Delta S_{12}} + \underbrace{6{,}914}_{\Delta S_{23}} + \underbrace{(-4{,}16)}_{\Delta S_{34}} + \underbrace{(-6{,}914)}_{\Delta S_{41}} = 0$ #

Entonces aplicando el 1er principio tenemos que $\underbrace{\Delta U_{ciclo}}_{0} = Q_{ciclo} + W_{ciclo}$ ∴

∴ $Q_{ciclo} = -W_{ciclo}$. Comprobación :

$Q_{ciclo} = Q_{12} + Q_{23} + Q_{34} + Q_{41} = 3656 + 12155 + (-7312) + (-6078) = \boxed{2421\,cal = Q_{ciclo}}$

$W_{ciclo} = W_{12} + W_{23} + W_{34} + W_{41} = 0 + (-4843) + 0 + 2422 = \boxed{-2421\,cal = W_{ciclo}}$

Por lo tanto tenemos una máquina térmica (motor) que nos proporciona 2421 cal de trabajo mecánico en cada ciclo. La pregunta es, ¿del calor absorbido por el gas, qué fracción se transforma en trabajo?

$\eta = \left| \frac{W_{ciclo}}{\sum Q_{absorbido}} \right| = \frac{2421\,cal}{(\underbrace{3656}_{Q_{12}} + \underbrace{12155}_{Q_{23}})\,cal} = 0{,}1531 \times 100 \Rightarrow \boxed{\begin{pmatrix} \text{rendimiento} \\ \text{del ciclo} \end{pmatrix} = 15{,}3\,\%}$

⇐ el gas solo absorbe calor en los procesos 1→2 y 2→3.