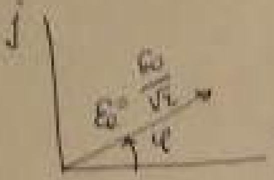
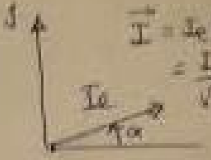
$\mathcal{E} = -\frac{d\phi}{dt} = B\cdot S\cdot\omega\cdot \operatorname{sen}\omega t \;\therefore\; \mathcal{E} = \mathcal{E}_0 \operatorname{sen}(\omega t + \varphi) \qquad \omega = \frac{2\pi}{T} = 2\pi\nu$



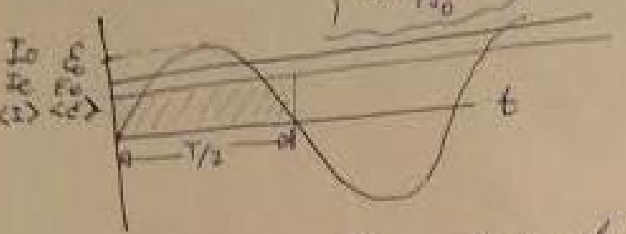
$\vec{\mathcal{E}} = \mathcal{E}_e \underline{|\varphi} = \frac{\mathcal{E}_0}{\sqrt{2}} \underline{|\varphi}$

Valor complejo de $\mathcal{E}$ e I

$\boxed{\begin{array}{l} \mathcal{E}_e = \sqrt{\langle \mathcal{E}^2 \rangle} \\ \langle \mathcal{E} \rangle = \frac{1}{T}\int_0^{T/2} \mathcal{E}\, dt \end{array}}$



$\vec{I} = I_e \underline{|\alpha} = \frac{I_0}{\sqrt{2}} \underline{|\alpha}$



$\mathcal{E}_e = \frac{\mathcal{E}_0}{\sqrt{2}} = 0{,}707\, \mathcal{E}_0$

$I_e = \frac{I_0}{\sqrt{2}} = 0{,}707\, I_0$

[redacted] Factor de amplitud de una magnitud sinusoidal $= \frac{I_0}{I_e} = \frac{\mathcal{E}_0}{\mathcal{E}_e} = \sqrt{2} = 1{,}4142$

$\langle \mathcal{E} \rangle = \frac{2}{\pi}\mathcal{E}_0 = 0{,}637\, \mathcal{E}_0$

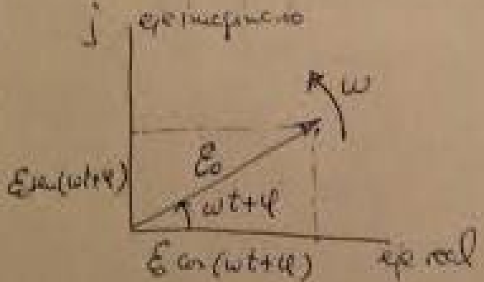
$\langle I \rangle = \frac{2}{\pi} I_0 = 0{,}637\, I_0$

Factor de forma de una magnitud sinusoidal $= \frac{I_e}{\langle I \rangle} = \frac{\mathcal{E}_e}{\langle \mathcal{E} \rangle} = \frac{\pi}{2\sqrt{2}} = 1{,}1107$

f.e.m. instantánea $\Rightarrow$ nº complejo giratorio (fasor)

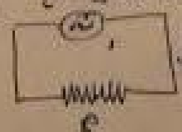
$\mathcal{E} = \mathcal{E}_0 \operatorname{sen}(\omega t + \varphi) = \operatorname{Im} \vec{\mathcal{E}}$

- binómica: $\vec{\mathcal{E}} = \mathcal{E}_0 \cos(\omega t + \varphi) + j\, \mathcal{E}_0 \operatorname{sen}(\omega t + \varphi)$
- polar: $\vec{\mathcal{E}} = \mathcal{E}_0 \underline{|\omega t + \varphi}$



CIRCUITO RESISTIVO PURO

$\vec{\mathcal{E}} = \mathcal{E}_e \underline{|\varphi}$ $\quad \mathcal{E} = \mathcal{E}_0 \operatorname{sen}(\omega t + \varphi)$



Ley de Ohm.

$\mathcal{E} = I \underset{\text{eje real}}{R}$

$I = \frac{\mathcal{E}}{R} = \underbrace{\frac{\mathcal{E}_0}{R}}_{I_0} \operatorname{sen}(\omega t + \varphi) = I_0 \operatorname{sen}(\omega t + \varphi)$

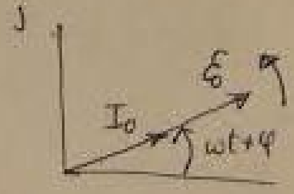
En forma compleja:

$\mathcal{E} = \mathcal{E}_0 \operatorname{sen}(\omega t + \varphi) \longrightarrow \vec{\mathcal{E}} = \mathcal{E}_e \underline{|\varphi}$

$I = I_0 \operatorname{sen}(\omega t + \varphi) \longrightarrow \vec{I} = I_e \underline{|\varphi}$

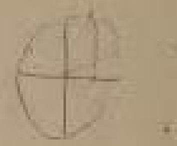
$\vec{R} = \frac{\vec{\mathcal{E}}}{\vec{I}} = \frac{\mathcal{E}_e \underline{|\varphi}}{I_e \underline{|\varphi}} = \frac{\mathcal{E}_e}{I_e} \underline{|0^\circ} = \frac{\mathcal{E}_0/\sqrt{2}}{I_0/\sqrt{2}} \underline{|0^\circ} = R \underline{|0^\circ}$

La fem y la tensión están en fase, pues ambos varían sinusoidalmente en $\omega t+\varphi$



CIRCUITO INDUCTIVO PURO

$$V = L\frac{dI}{dt} \; ; \; dI = \frac{1}{L} V dt$$



$$dI = \frac{1}{L} V_0 \operatorname{sen}\omega t \, dt$$

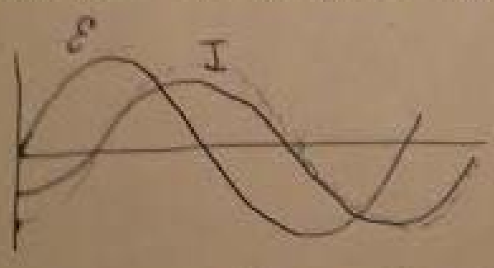
$$I = \frac{1}{L} V_0 \int_0^t \operatorname{sen}\omega t \, dt = \frac{V_0}{\omega L}(-\cancel{\cos\omega t})$$

$$-\operatorname{sen}(90^\circ - \omega t) = $$
$$= \operatorname{sen}(\omega t - 90^\circ)$$

$$\boxed{I = \frac{V_0}{\omega L}\operatorname{sen}\left(\omega t - \frac{\pi}{2}\right)}$$

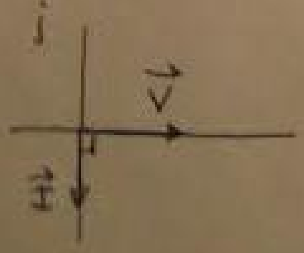
$I_0 = \frac{V_0}{\omega L}$ ← ωL juega el papel de una resistencia ⇒ reactancia inductiva o inductancia, $X_L = \omega L$ (Ω)

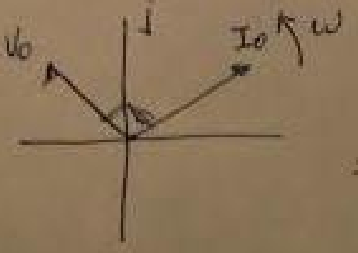
La intensidad está retrasada respecto a la tensión aplicada, 90°



↑X_L ⇒ ↑oposición al paso de la corriente

Valores complejos: $\vec{I} = I_e \angle -90^\circ$; $\vec{V} = V_e \angle 0^\circ$

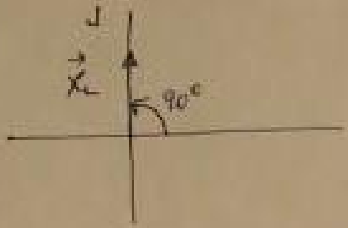




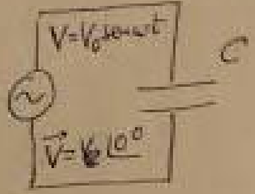
$$\frac{\vec{V}}{\vec{I}} = \frac{V_e \angle 0^\circ}{I_e \angle -90^\circ} =$$

$$= \frac{V_0/\sqrt{2}}{I_0/\sqrt{2}} \angle 90^\circ = X_L \angle 90^\circ$$

Induttanza complessa $\vec{X}_L = X_L \angle 90°$; $\vec{X}_L = jX_L = j\omega L$ $\Big\}$ $\vec{X}_L = \dfrac{\vec{V}}{\vec{I}}$



CIRCUITO CAPACITIVO PURO.



$$V = \frac{q}{C} \to q = CV \therefore \frac{dq}{dt} = C\frac{dV}{dt}$$

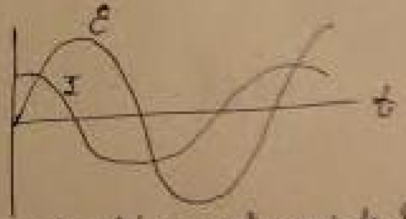
$$I = C\frac{dV}{dt} = C\omega V_0 \cos\omega t = \omega C V_0 \operatorname{sen}(\omega t + 90°)$$

$$I = \underbrace{\omega C V_0}_{I_0 = \frac{V_0}{\frac{1}{\omega C}}} \operatorname{sen}\left(\omega t + \frac{\pi}{2}\right)$$

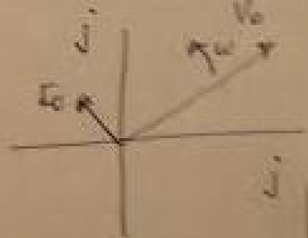
reattanza capacitiva o capacitanza: $X_C = \dfrac{1}{\omega C}$ (Ω)

$\vec{I} = I_e \angle 90°$

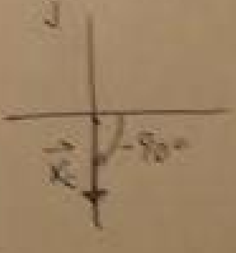
$\vec{V} = V_e \angle 0°$



I adelante 90° rispetto da E



$$\frac{\vec{V}}{\vec{I}} = \frac{V_e \angle 0°}{I_e \angle 90°} = \frac{V_0/\sqrt{2}}{I_0/\sqrt{2}} \angle -90° = X_C \angle -90°$$



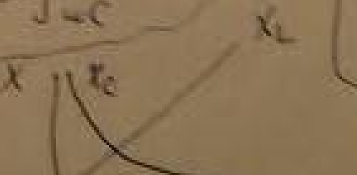
Capacitanza complessa: polare $\vec{X}_C = X_C \angle -90°$

binomica $\vec{X}_C = -jX_C = -j\dfrac{1}{\omega C}$

$\vec{X}_C = \dfrac{\vec{V}}{\vec{I}}$

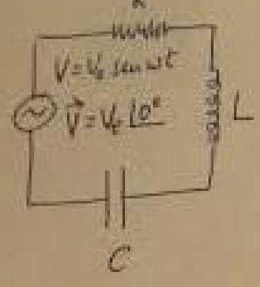
$$X_L = \omega L = 2\pi\nu L$$

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi\nu C}$$



Circuito RLC serie. Asociación de impedancias

En cada instante: $V = V_R + V_L + V_C$



$$V_R = IR$$

$$V_L = L\frac{dI}{dt}$$

$$V_C = \frac{q}{C} = \frac{1}{C}\int I\,dt$$

$$\mathcal{E}_0 \operatorname{sen}\omega t = IR + L\frac{dI}{dt} + \frac{1}{C}\int I\,dt$$

La solución es de la forma $I = I_0 \operatorname{sen}(\omega t - \varphi)$ ← sinusoidal

$$\frac{1}{C}\int I\,dt = \frac{-I_0}{\omega C}\cos(\omega t - \varphi)$$

$$\frac{dI}{dt} = I_0\omega\cos(\omega t - \varphi)$$

$$\mathcal{E}_0 \operatorname{sen}\omega t = I_0\operatorname{sen}(\omega t-\varphi)\,R + L I_0 \omega\cos(\omega t - \varphi) - \frac{I_0}{\omega C}\cos(\omega t - \varphi)$$

$$\mathcal{E}_0 \operatorname{sen}\omega t = R I_0\left(\operatorname{sen}\omega t\cos\varphi - \operatorname{sen}\varphi\cos\omega t\right) + \omega L I_0\left(\cos\omega t\cos\varphi + \operatorname{sen}\omega t\operatorname{sen}\varphi\right) -$$

$$- \frac{I_0}{\omega C}\left(\cos\omega t\cos\varphi + \operatorname{sen}\omega t\operatorname{sen}\varphi\right)$$

Es decir,

$$\mathcal{E}_0 \operatorname{sen}\omega t = \left[R\cos\varphi + \left(L\omega - \frac{1}{\omega C}\right)\operatorname{sen}\varphi\right] I_0 \operatorname{sen}\omega t - \left[R\operatorname{sen}\varphi - \left(L\omega - \frac{1}{\omega C}\right)\cos\varphi\right]$$

$$\mathcal{E}_0 = \left[R\cos\varphi + \left(\omega L - \frac{1}{\omega C}\right)\operatorname{sen}\varphi\right] I_0 \quad (1)$$

$$0 = R\operatorname{sen}\varphi - \left(\omega L - \frac{1}{\omega C}\right)\cos\varphi \quad (2) \Rightarrow \boxed{tg\,\varphi = \frac{\omega L - \frac{1}{\omega C}}{R}}$$

$$\left.\begin{array}{l} X_L = \omega L \\ X_C = \frac{1}{\omega C} \end{array}\right\} \boxed{tg\,\varphi = \frac{X_L - X_C}{R}}$$

$\operatorname{tg}\varphi = \dfrac{\omega L - \frac{1}{\omega C}}{R}$

De (1) $\Rightarrow \dfrac{\mathcal{E}_0}{I_0} = \left[R + \left(\omega L - \dfrac{1}{\omega C}\right)\dfrac{\sin\varphi}{\cos\varphi}\right]\cos\varphi = \left[R + \dfrac{\left(\omega L - \frac{1}{\omega C}\right)^2}{R}\right]\cos\varphi$

$$\operatorname{tg}^2\varphi = \frac{\sin^2\varphi}{\cos^2\varphi} = \frac{1-\cos^2\varphi}{\cos^2\varphi} = \frac{1}{\cos^2\varphi} - 1$$

$$\boxed{\cos\varphi = \frac{\mathcal{E}_0}{I_0}\,\frac{R}{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

$$\frac{\omega L - \frac{1}{\omega C}}{R^2} = \frac{I_0^2\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]^2}{\mathcal{E}_0^2 R^2} - 1$$

$$\omega L - \frac{1}{\omega C} + 1 = \frac{I_0^2}{\mathcal{E}_0^2}\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]^2$$

$$\cos^2\varphi = \frac{1}{1+\operatorname{tg}^2\varphi} \; ; \; \frac{\mathcal{E}_0^2}{I_0^2}\,\frac{\cancel{R^2}}{\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]^2} = \frac{\cancel{R^2}}{\cancel{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

$$1 + \operatorname{tg}^2\varphi = 1 + \frac{\left(\omega L - \frac{1}{\omega C}\right)^2}{R^2} = \frac{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}{R^2}$$

$$\boxed{\mathcal{E} = I_0\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

$$\boxed{I_0 = \frac{\mathcal{E}_0}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} = \frac{\mathcal{E}_0}{\sqrt{R^2 + (X_L - X_C)^2}}}$$

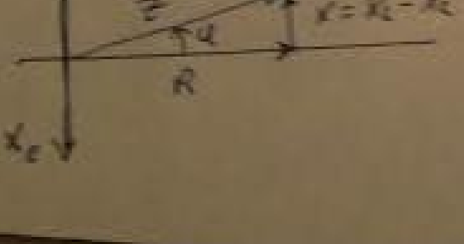
Z = impedancia $= \sqrt{R^2 + (X_L - X_C)^2}$

$\boxed{I_0 = \dfrac{V_0}{Z}}$ $\quad \operatorname{tg}\varphi = \dfrac{X}{R}$

X = reactancia $= \omega L - \dfrac{1}{\omega C} = X_L - X_C$

$Z = \sqrt{R^2 + X^2}$

triángulo de la impedancia



$$\vec{Z} = \frac{\vec{V}}{\vec{I}} = \frac{V_0\,[illegible]}{I_0\,[illegible]}$$

φ = fase V − fase I

$(\operatorname{arg} Z = \operatorname{arg} V / \operatorname{arg} I)$

$X_L > X_C \Rightarrow \varphi > 0 \Rightarrow I$ retrasada respecto a la tensión (circuito inductivo)

$X_L < X_C \Rightarrow \varphi < 0 \Rightarrow I$ adelantada " " " (circuito capacitivo)

$$\vec{Z} = \vec{R} + \vec{X_L} + \vec{X_C} = \vec{R} + \vec{X}$$

$\vec{Z}$: impedancia compleja

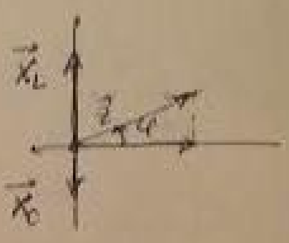
$\vec{X}$: reactancia compleja

$$\vec{Z} = \frac{\vec{V}}{\vec{I}} = \frac{V_0 \angle 0^\circ}{I_0 \angle -\varphi} = Z \angle \varphi$$

$$\boxed{\begin{array}{ll} \vec{Z} = Z \angle \varphi & \text{(forma polar)} \\ \vec{Z} = R + jX & \text{(forma binómica)} \end{array}}$$

Ley de Ohm en forma compleja

$$\vec{V} = \vec{I} \cdot \vec{Z}$$



Admitancia $= Y = \dfrac{1}{Z}$

Admitancia compleja: $\vec{Y} = \dfrac{1}{\vec{Z}} = \dfrac{1}{Z} \angle -\varphi$

$$\vec{Y} = \frac{1}{\vec{Z}} = \frac{1}{R + jX} = \frac{R - jX}{R^2 + X^2} = \frac{R}{R^2 + X^2} + j\,\frac{(-X)}{R^2 + X^2}$$

ELEMENTO	IMPEDANCIA, Z	ÁNGULO DE FASE, φ
R	R	0°
L	$X_L = \omega L$	$+90^\circ$ (I retrasada)
C	$X_C = \dfrac{1}{\omega C}$	-90° (I adelantada)
RL (serie)	$\sqrt{R^2 + X_L^2}$	$\operatorname{arc\,tg}\left(+\dfrac{\omega L}{R}\right)$
RC (serie)	$\sqrt{R^2 + X_C^2}$	$\operatorname{arc\,tg}\left(-\dfrac{1}{\omega C R}\right)$
LC (serie)	$X_L - X_C$	$\pm 90^\circ$
RLC (serie)	$\sqrt{R^2 + (X_L - X_C)^2}$	$\operatorname{arc\,tg}\left(\dfrac{\omega L - \frac{1}{\omega C}}{R}\right)$

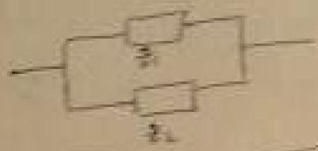
$$\left.\begin{array}{l}\dot{\vec{V}} = V_e \angle 0^\circ \\ \vec{I} = I_e \angle -\varphi\end{array}\right\} \vec{Z} = Z \angle \varphi \qquad \operatorname{tg}\varphi = \frac{X}{R} \; ; \; V = I Z$$

Asociación de impedancias

Serie

$$\vec{Z} = \vec{Z}_1 + \vec{Z}_2$$

Paralelo



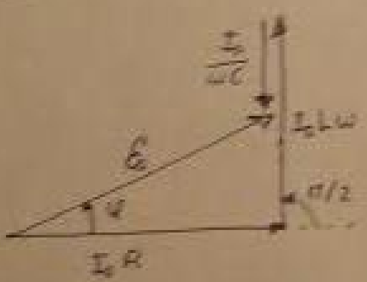
$$\frac{1}{\vec{Z}} = \frac{1}{\vec{Z}_1} + \frac{1}{\vec{Z}_2}$$

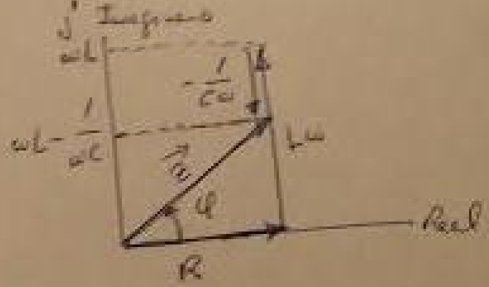
$$\vec{Y} = \frac{1}{\vec{Z}} = G + jS$$

$$G = \frac{R}{R^2 + X^2} = \frac{R}{Z^2} \quad \text{(conductancia)}$$

$$S = \frac{-X}{R^2 + X^2} = -\frac{X}{Z^2} \quad \text{(susceptancia)}$$

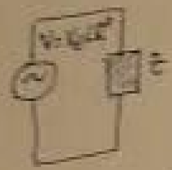
Construcción de Fresnel para un RLC





Potencia de la corriente alterna

$$P = V \cdot I$$



$$\left.\begin{aligned} V &= V_0 \operatorname{sen} \omega t \\ I &= I_0 \operatorname{sen}(\omega t - \varphi) \end{aligned}\right\} P = V_0 I_0 \operatorname{sen} \omega t \cdot \operatorname{sen}(\omega t - \varphi)$$

$$\cos(\alpha + \beta) = \cos\alpha \cos\beta - \operatorname{sen}\alpha \operatorname{sen}\beta$$

$$\cos(\alpha - \beta) = \cos\alpha \cos\beta + \operatorname{sen}\alpha \operatorname{sen}\beta$$

$$\cos(\alpha + \beta) - \cos(\alpha - \beta) = -2 \operatorname{sen}\alpha \operatorname{sen}\beta$$

$$\boxed{\cos A - \cos B = -2 \operatorname{sen}\frac{A+B}{2} \operatorname{sen}\frac{A-B}{2}}$$

$$\left.\begin{aligned} \alpha + \beta &= A \\ \alpha - \beta &= B \end{aligned}\right\} \rightarrow \left\{\begin{aligned} \alpha &= \frac{A+B}{2} \\ \beta &= \frac{A-B}{2} \end{aligned}\right.$$

$$\left.\begin{aligned} \frac{A+B}{2} &= \omega t \\ \frac{A-B}{2} &= \omega t - \varphi \end{aligned}\right\} \begin{aligned} A+B &= 2\omega t \\ A-B &= 2\omega t - 2\varphi \end{aligned} \rightarrow \left\{\begin{aligned} A &= 2\omega t - \varphi \\ B &= \varphi \end{aligned}\right\} \therefore$$

$$\therefore P = -\frac{1}{2} V_0 I_0 \left[\cos(2\omega t - \varphi) - \cos\varphi\right] = V_0 I_0 \left[-\cos(2\omega t - \varphi) + \cos\varphi\right]$$

$$\langle P \rangle = \frac{1}{T}\int_0^T P \, dt = \frac{V_0 I_0}{T}\int_0^T \left[-\cos(2\omega t - \varphi) + \cos\varphi\right] dt =$$

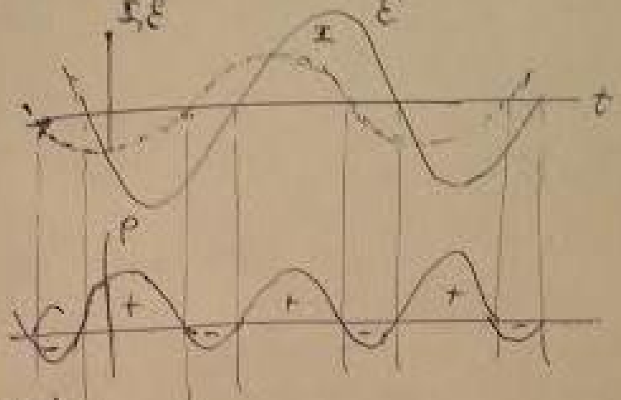
$x = 2\omega t - \varphi \quad dx = 2\omega \, dt$

$P = IV = I_0V_0 \sin(\omega t-\varphi)\sin\omega t = I_0V_0\left(\sin^2\omega t\cos\varphi - \frac{1}{2}\, 2\sin\omega t\cos\omega t\sin\varphi\right)$

$\sin(\omega t-\varphi) = \sin\omega t\cos\varphi - \sin\varphi\cos\omega t$

$\langle P\rangle = I_0V_0\left\langle \underbrace{\sin^2\omega t}_{\frac{1}{2}}\cos\varphi - \frac{1}{2}\underbrace{\sin 2\omega t}_{0}\sin\varphi\right\rangle$; $\langle P\rangle = \frac{I_0}{\sqrt{2}}\frac{V_0}{\sqrt{2}}\cos\varphi$

$$\boxed{\langle P\rangle = \underbrace{I_e V_e}_{\text{potencia aparente}} \underbrace{\cos\varphi}_{\text{factor de potencia}}}$$



$\langle P\rangle_{máx} \Rightarrow \varphi = 0$ ($\nexists$ L y C)

$\varphi = \pm\frac{\pi}{2} \Rightarrow \langle P\rangle = 0 \Rightarrow$ corriente sin vatios $\left\{\begin{array}{l}\text{sólo } C\\ \text{sólo } L\end{array}\right\}$ Entregada en $\frac{1}{4}T$ cede al generador en el siguiente $\frac{1}{4}T$

R=0 (nunca se da en la práctica)

$\cos\varphi = 1 \Rightarrow$ Resonancia

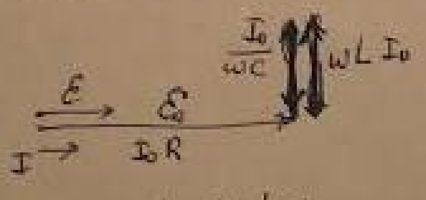
Circuito resonante RCL $\therefore$ $Z = \sqrt{R^2 + \left(L\omega - \frac{1}{\omega C}\right)^2}$

$Z_{mín} = R$ cuando $\omega L - \frac{1}{\omega C} = 0 \Rightarrow$

$\Rightarrow \left\{\begin{array}{l}\text{Amplitud de la intensidad } I_0 = \frac{\mathcal{E}_0}{Z};\ I_0^{máx} = \frac{\mathcal{E}_0}{R}\\ \varphi = 0 \Rightarrow \cos\varphi = 1\end{array}\right.$

I en fase con V

$I_{0máx}$ es la misma que DC



$\omega_0^2 = \frac{1}{LC}$; $\omega_0 = \frac{1}{\sqrt{LC}} = 2\pi\nu_0$

Gráficamente $X_L = \omega L = 2\pi L\nu$

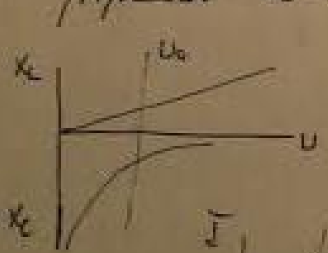
$-X_C = \frac{-1}{\omega C} = \frac{-1}{2\pi\nu C}$

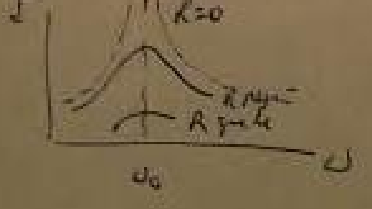
$$\boxed{\nu_0 = \frac{1}{2\pi\sqrt{LC}}}$$

$\omega < \omega_0 \Rightarrow$ capacitivo

$\omega > \omega_0 \Rightarrow$ inductivo

$R \ll X_L$ ó X_C





En la práctica $\cos\varphi \simeq 0{,}85$ $\langle P\rangle$ menor que P_a

$$\boxed{\langle P\rangle = I_e^2 R \cos\varphi = \left(\frac{\varepsilon_e^2}{Z}\right)\cos\varphi} \boxed{= I_e V_e \cos\varphi}$$

puede ser reducimos I_e ⇒ ↓ consumo de E tan pequeño como queramos siempre que $R\downarrow\downarrow\downarrow$.

Potencia compleja $\vec{S}$

$\vec{V} = V_e \angle 0^\circ$

$\vec{I} = I_e \angle -\varphi$

$\vec{Z} = Z\angle\varphi$

$$\vec{S} = \vec{V}\cdot\vec{I}^* = V_e\angle 0 \cdot I_e\angle\varphi = V_e I_e \angle\varphi$$

$$\vec{S} = V_e I_e \angle\varphi$$

$$\vec{S} = P + jQ$$

TRIÁNGULO DE POTENCIAS

potencia aparente (VA)

$S = V_e I_e$

$Q = V_e I_e \operatorname{sen}\varphi$ ⇒ potencia reactiva (VAR)

φ → desfase entre I y V

$P = V_e I_e \cos\varphi$ potencia activa (W)

↑ factor de potencia $\cos\varphi = \dfrac{P}{S}$

S = potencia aparente (R) VA

⇒ $I_e = \dfrac{P}{V_e\cos\varphi}$, $S = V_e I_e$ } $\downarrow\cos\varphi \Rightarrow \uparrow I_e, \uparrow S$ ⇒ cables gruesos, generadores mayores, penalizan

$$S^2 = P^2 + Q^2$$

$$\vec{V} = \vec{I}\cdot\vec{Z}$$

$$\vec{S} = \vec{V}\cdot\vec{I}^* = (\vec{I}\cdot\vec{Z})\,\vec{I}^* = I_e\angle -\varphi \; Z\angle\varphi \; I_e\angle\varphi =$$

$$= I_e^2 Z\angle\varphi = I_e^2 R + j I_e^2 X$$

$P = I_e^2 R$

$Q = I_e^2 X$

CVIL { VIL ⇒ inductivo $\varphi>0$, V adelanta a I; CIV ⇒ capacitivo $\varphi<0$, I adelanta a V }

$I = I_0(\operatorname{sen}(\omega t - \varphi))$

$V = V_0 \operatorname{sen}\omega t$

⇒ $\varphi>0$ ⇒ inductivo I por detrás de V ⇒ fdp $= \cos\varphi$ en retraso (inductivo)

$\varphi<0$ ⇒ capacitivo I por delante de V ⇒ fdp $= \cos\varphi$ en adelanto (capacitivo)